

**Name:** \_\_\_\_\_

NOTES:

- This is a practice exam. It will not be graded. The aim is to give you a sense of the types of questions that may appear on the exam.
- As a reminder, this is not an exhaustive list of types of problems, anything through Turing Machines (class on October 1) is fair game.
- I recommend preparing your notes sheet before attempting these problems, then revising it to see if it needs to be adapted.
- Attempt all problems. On the exam, it is possible to receive partial credit for showing your work.

**Problem 1**

Score:

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Give a finite automata (deterministic or non-deterministic) that recognizes the following language over the alphabet  $\Sigma = \{0, 1\}$ :

$$L = \{w \mid w \text{ is an even length string with an odd number of 0s}\}.$$

**Problem 2**

Score:

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Let  $\text{flip}(L)$  be the language of strings drawn from alphabet  $\Sigma^* = \{0, 1\}$  such that if  $w \in L$  then  $w' \in \text{flip}(L)$ , where  $w'$  is  $w$  but every 0 is a 1 and every 1 is a 0.

For example if  $0100 \in L$  then  $1011 \in \text{flip}(L)$ .

Show that if  $L$  is regular, then  $\text{flip}(L)$  is also regular. You do not have to give a fully-symbolic proof, but your argument should cover all relevant details of the construction.

**Problem 3**

Score:

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Consider the following language over the alphabet  $\Sigma = \{ (, ) \}$ , i.e., open and close parentheses.

$$L = \{ w \mid w \text{ is a string of balanced parentheses} \}$$

Prove that  $L$  is irregular using any method you prefer.

**Problem 4**

Score:

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Consider the following context-free grammar  $G$  with start symbol  $S$ :

$$S \rightarrow XYX$$

$$X \rightarrow X0 \mid 0$$

$$Y \rightarrow 1Y \mid \epsilon$$

(a) Give a derivation demonstrating that  $S \xRightarrow{*} 00100$ .

(b) Give a formal definition for  $L(G)$ .

**Problem 5**

Score:

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Give a **textual** implementation-level description of a TM that recognizes the following language over  $\Sigma = \{0, 1\}$ .

$$L = \{w \mid w \text{ does not contain twice as many 1s as 0s}\}.$$

Make sure in your description to describe any pre-processing you do to the tape, *e.g.*, shifting and marking, as well as the precise movement of the head throughout the operation of the machine.

**Problem 6**

Score:

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Let a *Right-skip Turing Machine* (RTM) be a (deterministic) Turing Machine that operates like a normal Turing machine except that when it moves its head to the right, it moves *two* tape cells instead of just one. A RTM cannot move its tape head a single cell to the right in a single step of computation.

Prove that an RTM is equivalent to a TM. You do not have to give a fully symbolic account of your constructions, but your prose should cover all relevant details.

(*Hint: make sure to prove both sides of the equivalence! They are not symmetric constructions!*)